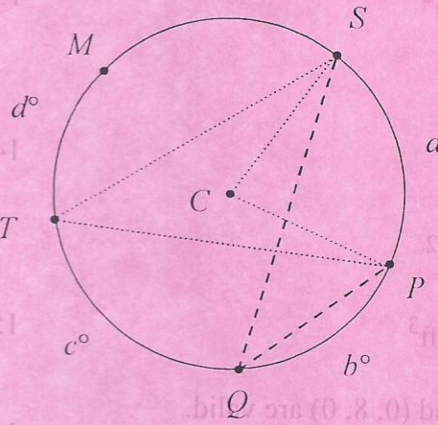
SOLUTIONS.

1. Find, in degrees the values of $m\angle PQS$ and $m\angle MQS$, given:

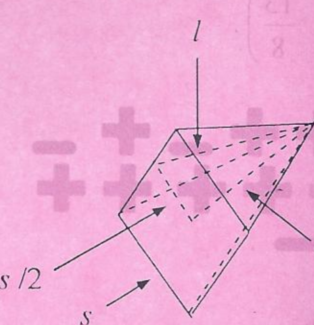
$a° = 72°$
$b° = 52°$
$c° = 82°$
$d° = 39°$



$m\angle PQS$ = ½ of $a°$ = 36°

$m\angle MQS$ = ½ of $m\angle MCS$
= ½ of (360 - $a°$ - $b°$ - $c°$ - $d°$)
= 57.5°

2. At an archeological site is a square-based pyramid. Each side of the base B has length s = 190 feet. The vertical height of the pyramid is h = 228 feet.



a. Area of base B = s^2 = 36100 ft^2

b. Lateral Area (L.A.) = ½ pl = $\frac{1}{2}(4s)\sqrt{(s/2)^2 + h^2}$ = $380\sqrt{61009}$ ft^2 = 93860 ft^2

c. Surface Area (S.A.) = L.A. + B = 93860 ft^2 + 36100 ft^2 = 129960 ft^2

d. Volume (V) = (1/3)·B·h = (1/3)· s^2·h = (1/3)· 36100·228 ft^3 = 2743600 ft^3

3. $x^2+(y-8)y=0 \quad \Rightarrow \quad \dfrac{x^2}{16}+\dfrac{(y-4)^2}{16}=1$ is a circular cylinder about (0, 4, *z*).

$y(8-y)=4z^2 \quad \Rightarrow \quad \dfrac{z^2}{4}+\dfrac{(y-4)^2}{16}=1$ is an ellipsoidal cylinder about (*x*, 4, 0).

The two cylinders intersect at (0, 0, 0) and (0, 8, 0). (A sketch is helpful.)

$4x^2+(y-4)^2+2z^2=16 \quad \Rightarrow \quad \dfrac{x^2}{4}+\dfrac{(y-4)^2}{16}+\dfrac{z^2}{8}=1$ is an ellipsoid.

Checking the previously found points, we find both (0, 0, 0) and (0, 8, 0) are valid.

4. Solve. Use only Rational Numbers to express your answer and reduce to lowest terms.

$2x+4y+8z=16$
$3x+9y+27z=81$
$4x+16y+64z^2=262$

From the first two equations: $y = -5 \cdot z + 19$, $x = 6 \cdot z - 30$.

Substituting these expressions into the third equation: $-56 \cdot z + 184 + 64 \cdot z^2 = 262$.

There are two solutions: $\left(-\dfrac{69}{2}, \dfrac{91}{4}, -\dfrac{3}{4}\right)$ and $\left(-\dfrac{81}{4}, \dfrac{87}{8}, \dfrac{13}{8}\right)$.

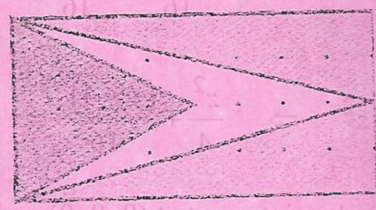
5. + + + + − + − remove + + + + + leaves − − .

6. One must find the arc length $s=\displaystyle\int_{x_0}^{x_f}\sqrt{1+(y')^2}\,dx$.

$y=\dfrac{1}{3}x^{3/2} \quad \Rightarrow \quad y'=\dfrac{1}{2}\sqrt{x} \quad \Rightarrow \quad s=\displaystyle\int_0^4 \tfrac{1}{2}\sqrt{x+4}\,dx = \left[\tfrac{1}{3}(x+4)^{3/2}\right]_0^4 = \dfrac{16}{3}\sqrt{2}-\dfrac{8}{3}$

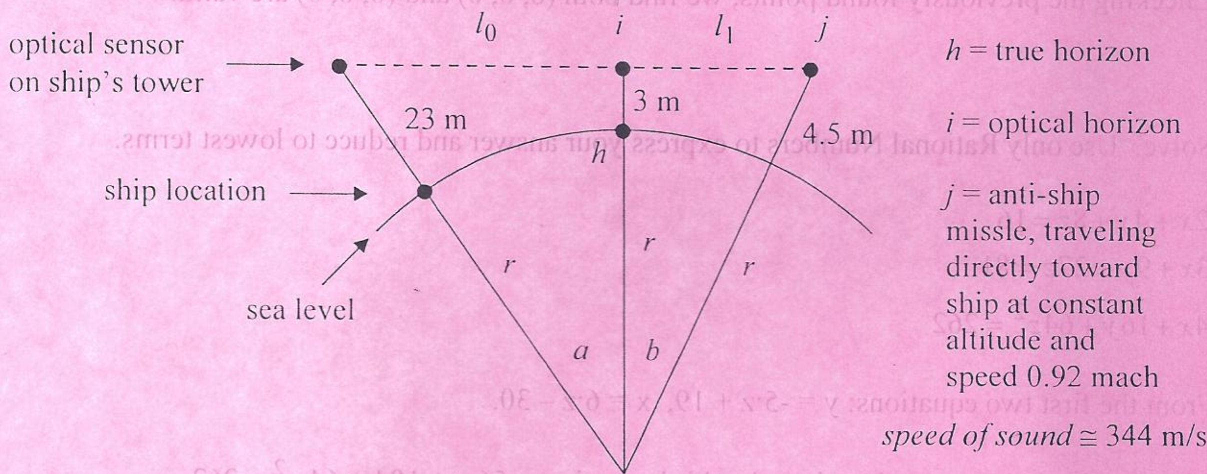
or $s \cong 4.88$ miles.

7. The two long horizontal triangles can be combined to "cover" the upper half of the rectangle. The remaining isosceles triangle can be split in half horizontally and the two pieces will cover half of the lower half of the rectangle. One fourth of the rectangle remains.



Shaded area = ¾ .

8. Sketch (not to scale !):



a. $l = l_0 + l_1 \cong 15958.69669 + 4370.354677 \cong 20329$ meters

b. length R = r + 3.

angle $a = arcsin(l_0 / \text{R}) \cong 0.2506471789\text{e-}2$

angle $b = arcsin(l_1 / \text{R}) \cong 0.6864069359\text{e-}3$

angle $c = a + b$.

length H = r + 4.5.

Circumference C = 2πH.

flight path $s = \dfrac{c}{2\pi} \cdot C \cong 20329.07321$ meters

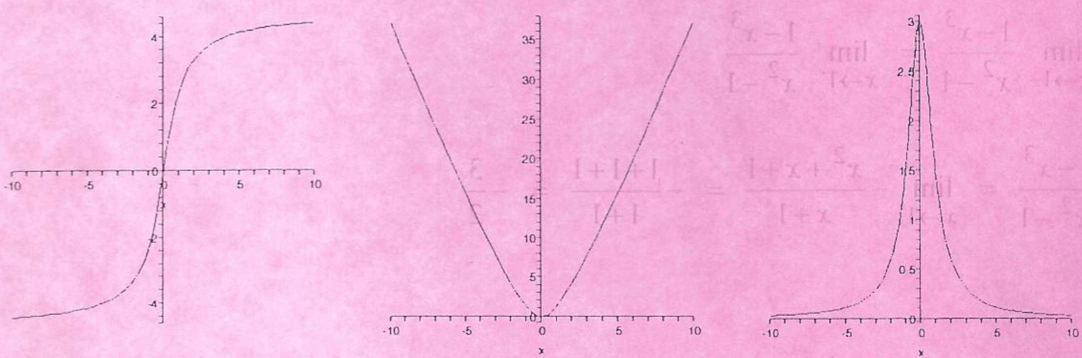
velocity $v = 0.92 \cdot 344$ m/s = 316.48 m/s.

$t = s / v \cong 64.24$ seconds (rounded from 64.2349).

9. Determine which # graph below matches the given functions.

a. $f(x)$ **#2** b. $\frac{d}{dx}f(x)$ **#1** c. $\frac{d^2}{dx^2}f(x)$ **#3**

#1 #2 #3



10. A mass is undergoing exponential decay at the constant rate of $\lambda = 0.0025$ per second. How many seconds will elapse before only ¼ of the mass is left?

Exponential decay: $m_f = m_0 \cdot e^{-\lambda t}$.

$\frac{1}{4} = e^{(-0.0025)t}$ implies $t \cong 554.5$ seconds.

11. Reduce $\frac{2+3i}{2-5i}$ to lowest rational terms, where $i \equiv \sqrt{-1}$.

$-\frac{11}{29} + \frac{16}{29}i$.

12. A standard deck of playing cards has 52 cards. There are 4 "suits" in one deck, each with an equal number of cards. A "Royal Flush" involves 5 cards from the same suit, and there is only one possible "royal flush" per suit.

A deck of cards is placed face down in random order. What is the odds of drawing the first five cards and receiving a Royal Flush?

There are "52 choose 5" = $\frac{52!}{5! \cdot (52-5)!}$ = 2,598,960 possible draws. There are only 4 possible "Royal Flushes".

The odds are 4 : 2598960.

13. Solve, if possible: $\lim_{x\to 1} \dfrac{1-x^3}{x^2-1}$

$$\frac{1-x^3}{x^2-1} = \frac{(1-x)(x^2+x+1)}{(x+1)(x-1)} = -\frac{x^2+x+1}{x+1} \quad \text{for } x \neq 1$$

Further: $\lim_{x\to 1^-} \dfrac{1-x^3}{x^2-1} = \lim_{x\to 1^+} \dfrac{1-x^3}{x^2-1}$

So $\lim_{x\to 1} \dfrac{1-x^3}{x^2-1} = \lim_{x\to 1} -\dfrac{x^2+x+1}{x+1} = -\dfrac{1+1+1}{1+1} = -\dfrac{3}{2}$

14. For this function: $f(x) = -3x^2 + 16x - 2x \cdot \sqrt{9x^2 + 6x + 1}$

a. What are the x-intercepts? ____________________
b. What are the y-intercepts? ____________________

For the x-intercepts, carefully examine $-3x^2 + 16x - 2x \cdot \sqrt{(3x+1)^2} = 0$

to find $x = 0, \dfrac{14}{9}, -6$.

For the y-intercepts, $f(0) = -3 \cdot 0^2 + 16 \cdot 0 - 20 \cdot \sqrt{9 \cdot 0^2 + 6 \cdot 0 + 1} = 0$.

15. Evaluate $\log_4\left(18^{25}\right)$ to an accuracy of 10^{-3}.

$\log_4\left(18^{25}\right) = 25 \cdot \log_4 18 = 25 \cdot \ln(18)/\ln(4) \cong 52.124.$

16. Find $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n}$

$s_1 = \dfrac{1}{2},\ s_2 = \dfrac{3}{4},\ s_3 = \dfrac{7}{8},\ \ldots,\ s_n = 1 - \dfrac{1}{2^n}.$ $\qquad \lim_{n\to\infty}\left(1 - \dfrac{1}{2^n}\right) = 1.$

17. Solve $x^2 - 2x - 3 > 1$

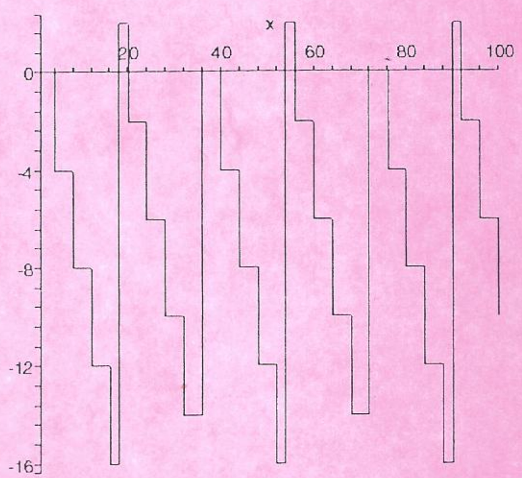
Examining the roots of $x^2 - 2x - 4 = 0$:

$x \in \left(-\infty,\ 1-\sqrt{5}\right) \cup \left(1+\sqrt{5},\ \infty\right)$

18. The *mod* function is defined: $\text{mod}_m(n) \equiv$ the remainder of $(n \div m)$ generally over the domain of integers. For example, $\text{mod}_5(7) = 2$.

Find the first integer $n > 4$ such that $\text{mod}_4(n) - \text{mod}_{18}(n) = 0$.

The first concurrence of 4 and 18 is at 36. A plot of the difference is:



19. Solve: $\begin{pmatrix} 1 & -2 \\ -3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$.

The matrix is singular. There is no solution.

20. Solve: $\left|\frac{1}{x-1}\right| < \left|\frac{1}{x}\right|$

$x \in (-\infty,\ 0) \cup \left(0,\ \frac{1}{2}\right)$.